Written calculations policy for Brookland Infant and Brookland Junior School

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

Aims

- **To outline for teaching staff and parents the written strategies for calculation taught at Brookland Infant and Junior School for addition, subtraction, multiplication and division, in line with the new curriculum for Mathematics.**
- **To show how using key pieces of practical maths apparatus helps to accelerate the children's learning.**
- **To ensure consistency of approach from one year group to the next.**
- **To enable children to develop confidence and fluency in calculations that they will be able to apply to a variety of problem-solving activities.**

At Brookland Infant and Junior schools, we aim, through creative and inclusive lessons, to create a sense of excitement and curiosity around Mathematics. Children are encouraged to make links between what they are learning and the world around them. A high quality maths education provides a foundation for understanding of the world. Maths is essential to everyday life and necessary in almost all forms of employment. As children learn mathematics, they are acquiring fluency in mental methods (maths they do in their heads) as well as written methods.

Although the way we teach calculation is organised in a sequence, teaching staff work with the ethos that individual children's needs denote the part of the curriculum that should be accessed. Progression in mathematics for all children is essential and so, no matter what their starting point, through accurate assessment, high expectations and quality teaching, pupils are able to realise their mathematical potential. All teachers ensure children with special educational needs are as carefully planned for and inclusivity is at the heart of what we do. Cross-curricular links are made where possible, particularly in science, through the use of technology and during whole school topics. Children at our schools understand that mathematics can be found everywhere and in everything, and exploring and being creative with maths is essential to developing an enthusiasm and fascination for the subject.

The National Curriculum for Mathematics aims to ensure all pupils:

- **Become fluent in fundamentals of mathematics so that they are efficient in using and selecting the appropriate written algorithms and mental methods, underpinned by mathematical concepts**
- **Can solve problems by applying their mathematics to a variety of problems with increasing sophistication, including in unfamiliar contexts and to model real-life scenarios**
- **Can reason mathematically by following a line of enquiry and develop and present a justification, including in unfamiliar mathematical language.**

At our schools, children are first given the opportunity to explore mathematical concepts using the following practical resources;

Numicon – aids children in recognising how much a number is worth, ordering and comparing numbers. Numicon is also used to help children explain their mathematical thinking when problem solving.

Counters – counters are used to aid children with counting. We also use the different colours to represent different amounts, introducing algebraic thinking.

Bead strings – bead strings are used for all four operations (addition, subtraction, division and multiplication). They are also used to count up in different amounts.

Dienes/base 10 – used to demonstrate a visual representation when understanding the value of numbers. They are also used for all four operations. Dienes/base 10 are related to each other in terms of size helping children to compare numbers.

Place value counters

These aid children with their understanding of place value when learning written methods of the four operations.

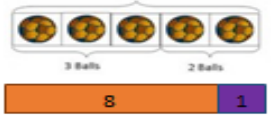
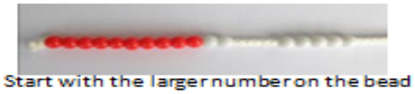
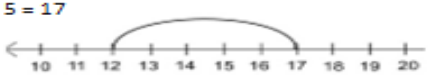
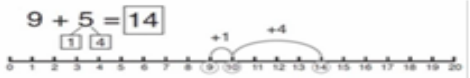
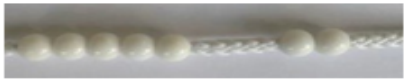
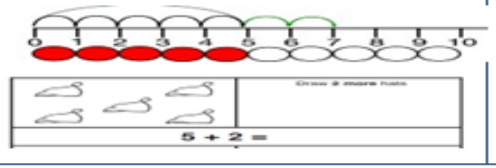
Place value table - aids children in understanding how much a number is worth. This is also used when multiplying and dividing by 10, 100 and 1000... showing how the digits move to the left or the right.

Hundred number square - used to aid early counting. Excellent for adding and subtracting 10. Children can use this to find patterns in multiplication tables.

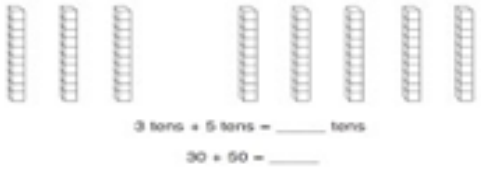
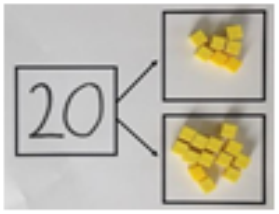
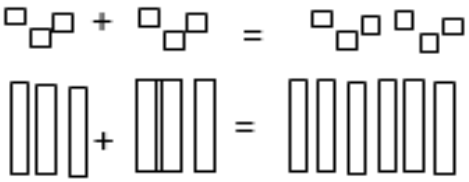
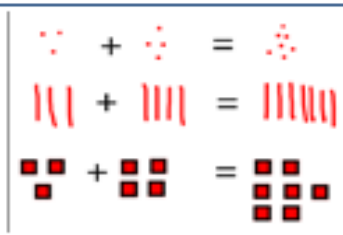
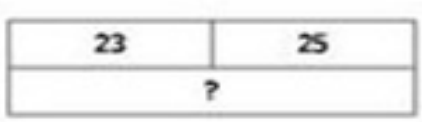
Place value arrow cards – used to demonstrate how much a number is worth. Also used to partition numbers (break up into, for example, hundreds, tens and units) and add or subtract.

Money – used when problem solving with money and to help understanding of decimals.

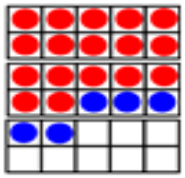
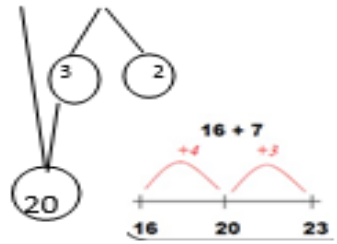
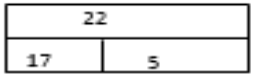
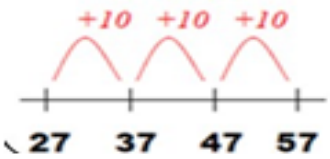
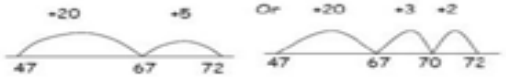
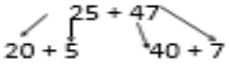
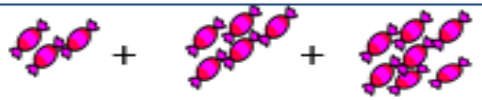
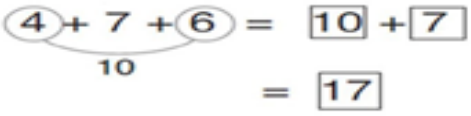
Year 1 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	  Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$  $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	6 + 5 = 11   Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.		Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

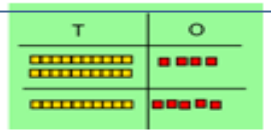
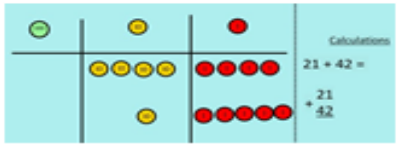
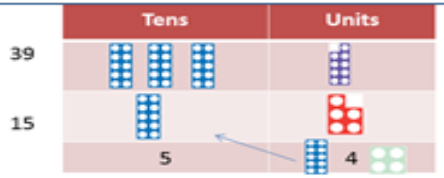
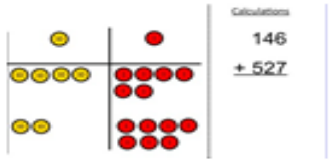
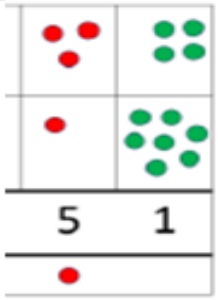
Year 1 Addition

Objective & Strategy	Concrete	Pictorial	
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

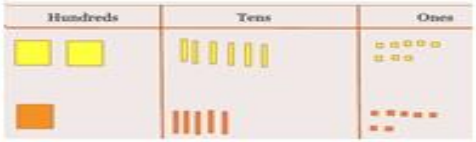
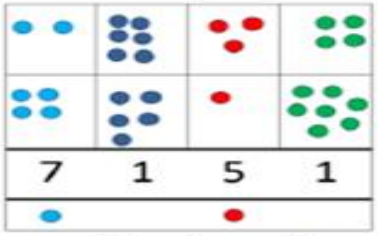
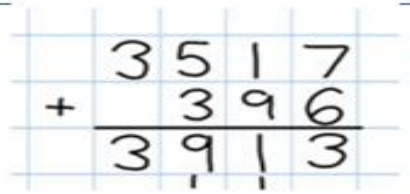
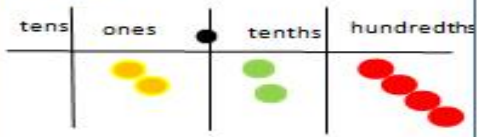
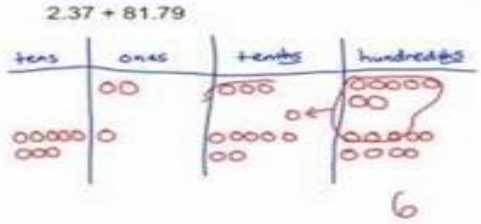
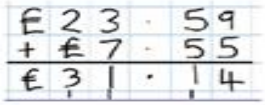
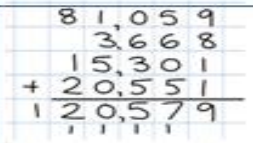
Year 2 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	27 + 30 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.

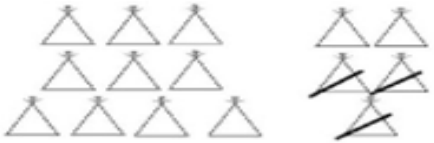
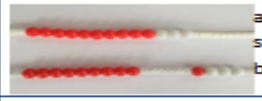
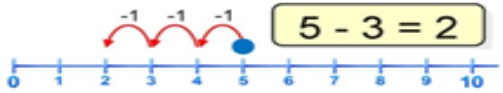
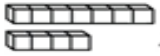
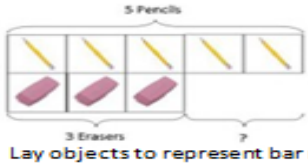
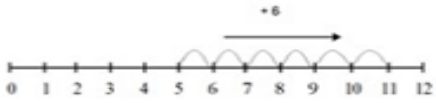
Year 3 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1

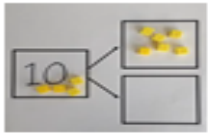
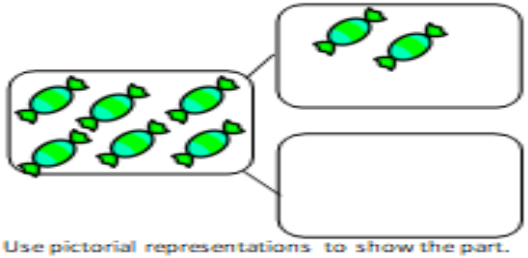
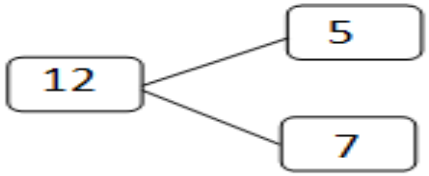
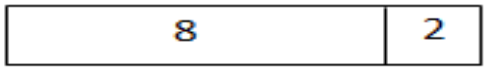
Years 4- 6 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		72.8 + 54.6 127.4 1 1 
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	 Insert zeros for place holders. 

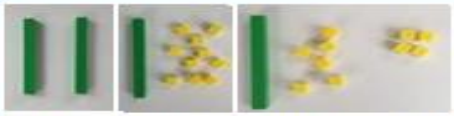
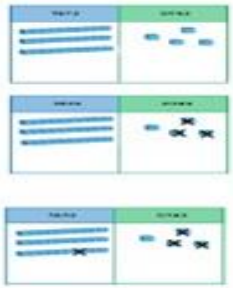
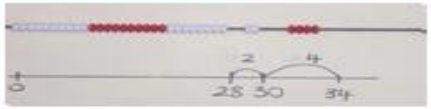
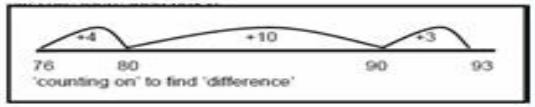
Year 1 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2	 15 – 3 = 12 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	  Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

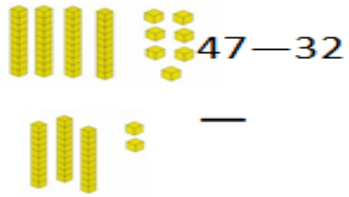
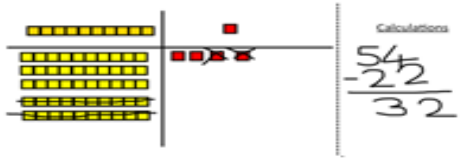
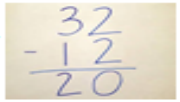
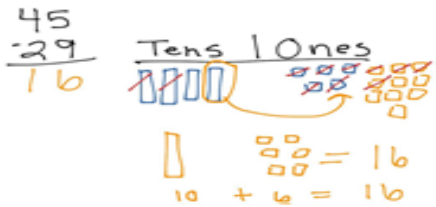
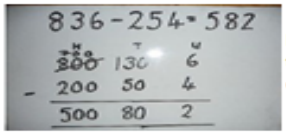
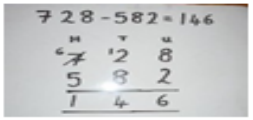
Year 1 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

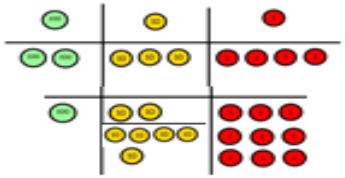
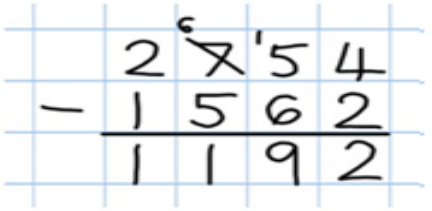
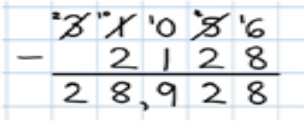
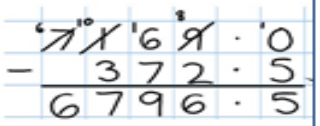
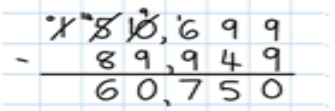
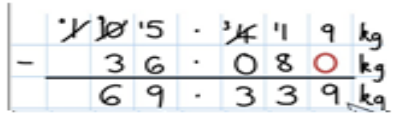
Year 1/2 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 20 — 4 =	20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

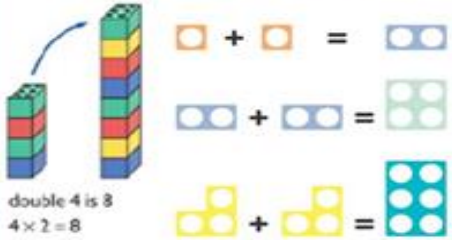
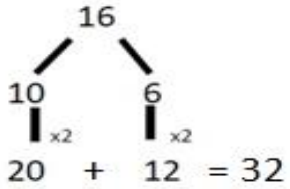
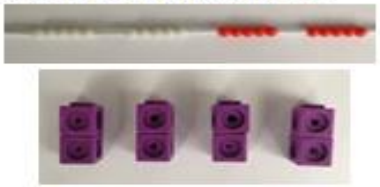
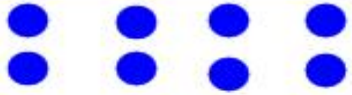
Year 3 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Darw representations to support understanding	$47-24=23$ $40+7$ $-\ 20+4$ $20+3$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

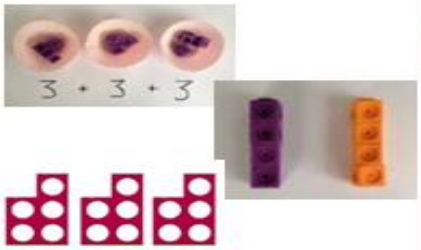
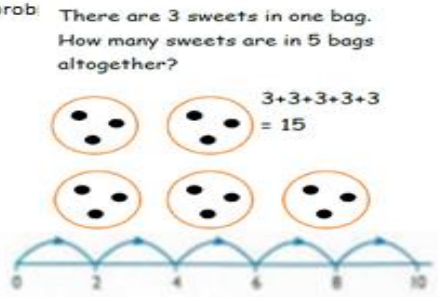
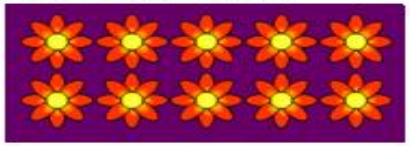
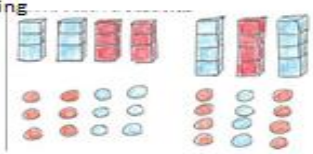
Years 4-6 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

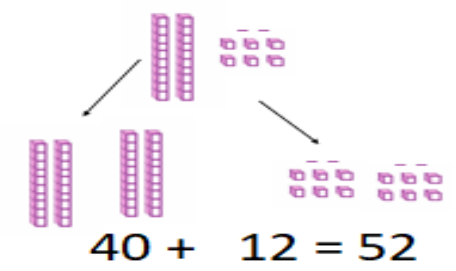
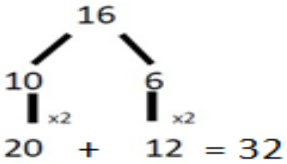
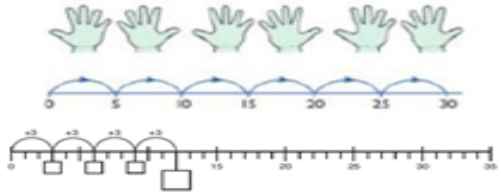
Year 1 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

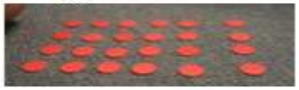
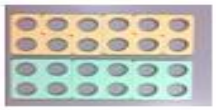
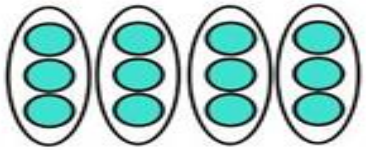
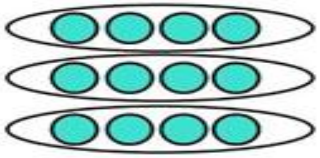
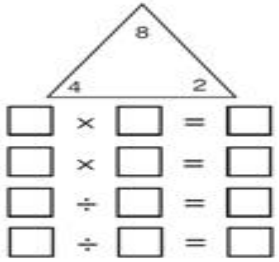
Year 1 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

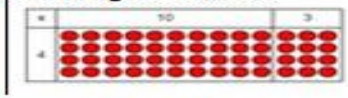
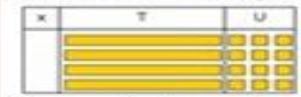
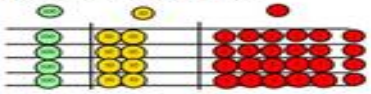
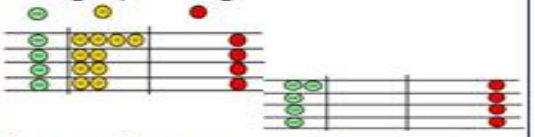
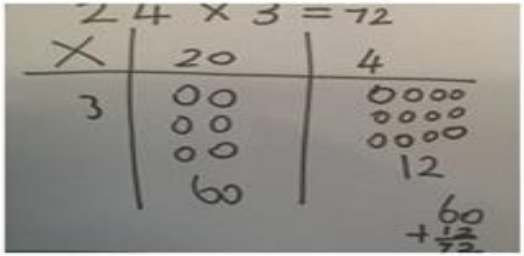
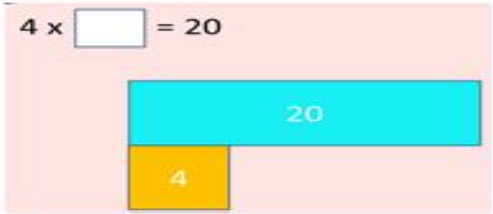
Year 1 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  16 10 ×2 6 ×2 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   ?	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

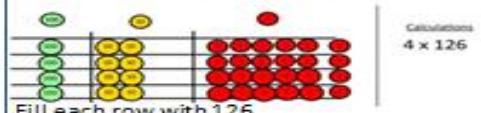
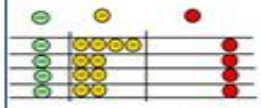
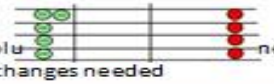
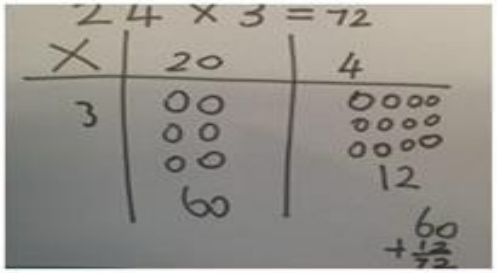
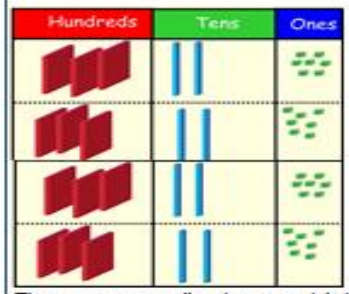
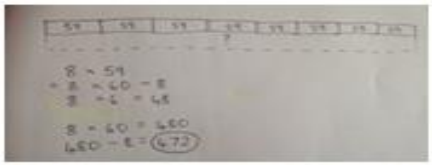
Year 1/2 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

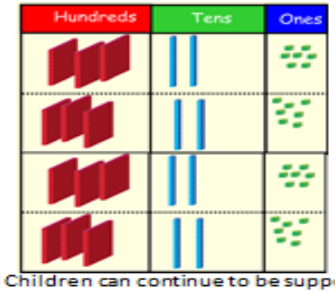
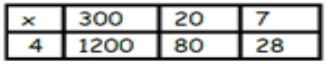
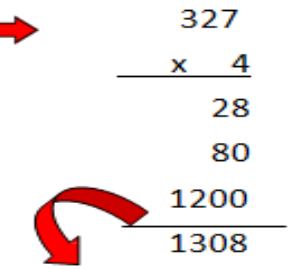
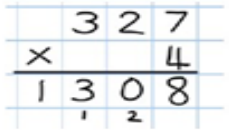
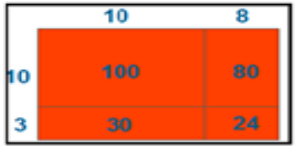
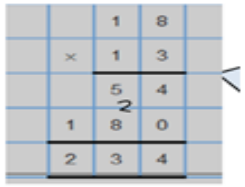
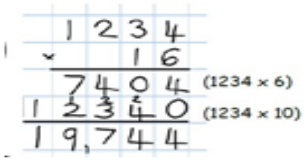
Year 3 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **×** | **30** | **5** **7** | **210** | **35** **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 10 | 8 10 | 100 | 80 3 | 30 | 24

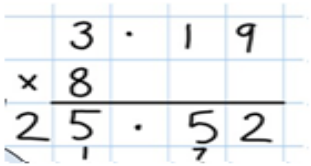
Year 4 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126   Add up each colu nes making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | x | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	| x | 300 | 20 | 7 | | --- | --- | --- | --- | | 4 | 1200 | 80 | 28 | The grid method my be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 3 2 7 x 4 1 3 0 8 1 2 This may lead to a compact method.

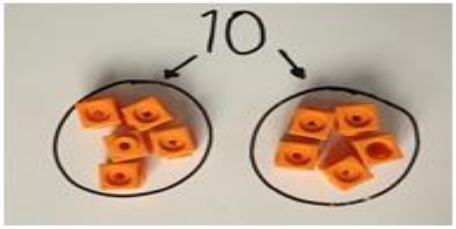
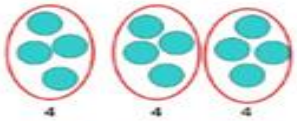
Years 5/6 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	 It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642		  This will lead to a compact method.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first 

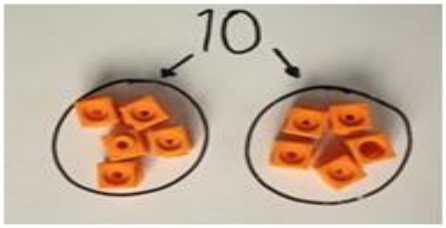
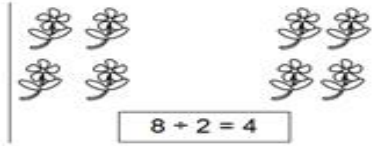
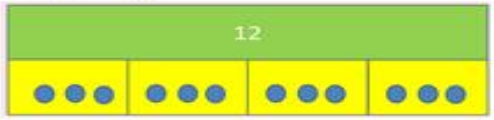
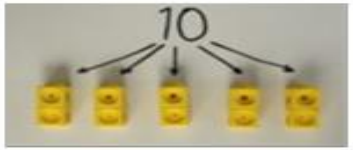
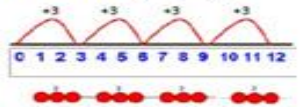
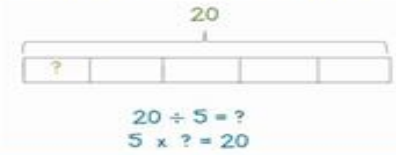
Year 6 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

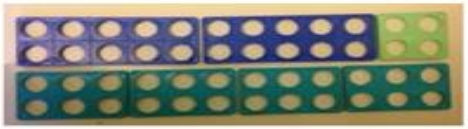
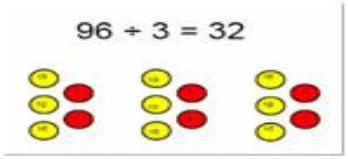
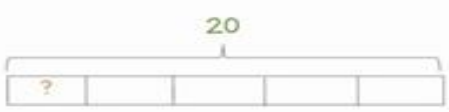
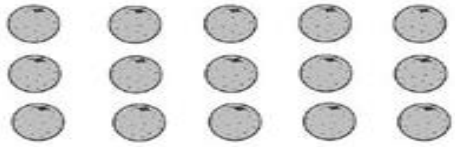
Year 1 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4	12 shared between 3 is 4

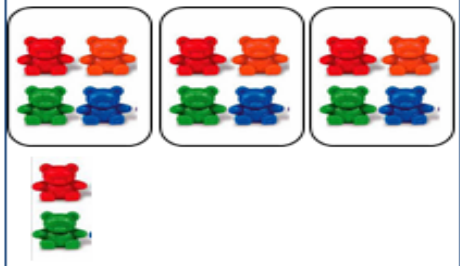
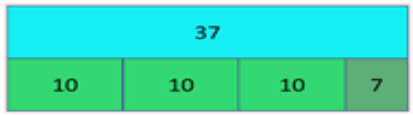
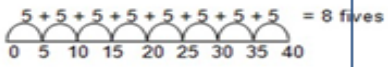
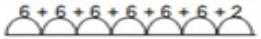
Year 2 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

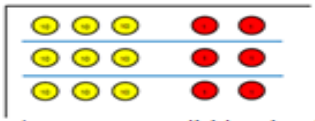
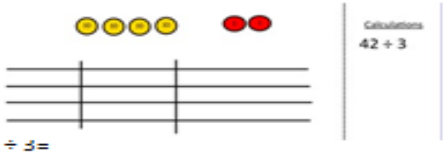
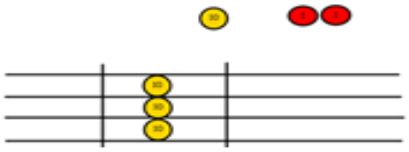
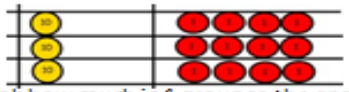
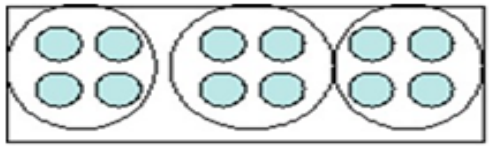
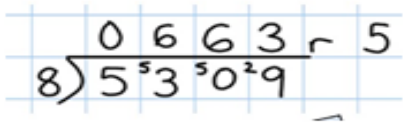
Year 3 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Year 3/4 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  Example without remainder: 40 ÷ 5 Ask "How many 5s in 40?"  = 8 fives 0 5 10 15 20 25 30 35 40 Example with remainder: 38 ÷ 6  = 6 sixes with a remainder of 2 0 6 12 18 24 30 36 38 For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.	Complete written divisions and show the remainder using r. 

Year 4/5 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 | 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 | 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 | 5 1 1 . 0  0 6 6 3 r 5 8 | 5 5 3 5 0 2 9

Year 6 Division

Long Division

Step 1—a remainder in the ones

```
    h  t  o
      0  4  1 R1
   ---------
4 ) 1  6  5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th h  t  o
        0  4  0  0 R7
   ------------
8 ) 3  2  0  7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Year 6 Division

Long Division

Step 1 continued...

h	t	o
	6	1
4)	2	4 7
	−	4
		3

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

th	h	t	o
	4	0	2
4)	1	6	0 9
		−	8
			1

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Year 6 Division

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Year 6 Division

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 -2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 There are no more digits to drop down. The quotient is 139.